Quantum Generative Adversarial Networks

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Abstract: *Generative adversarial networks have emerged as the primary candidate for generative machine learning. With the recent development of noisy intermediate-scale quantum devices it is pertinent to explore the potential advantages of quantum generative adversarial networks (QGANs). Here, we examine a significant challenge faced in previous implementations of a QGAN. Specifically, we propose a new method of dimensionality reduction of MNIST handwritten digits such that they could be encoded in a limited number of qubits. Rather than applying suboptimal principal component analysis, we employ a classical autoencoder to perform the required dimensionality reduction. We demonstrate this augmented QGAN still learns the underlying probability distribution while generating images qualitatively superior to previous QGANs. Accompanying code for this paper is available at https://github.com/hillspen/qgan.*

1. INTRODUCTION

1.1 Motivation

Generative adversarial networks (GANs) are deep neural networks capable of learning the distribution of a real dataset [1]. This is achieved by training two independent networks, the generator and discriminator, in competition with one another [2]. The discriminator aims to classify data as real or fake while the generator attempts fool the discriminator by creating fake data that mimics the real distribution [3]. Though the discriminator is exposed to real data as it learns to classify, the generator learns solely via its correspondence with the discriminator [2]. Mathematically, GAN training aims to minimize

$$\min_G \max_D \begin{matrix} \{E_{\mathbf{x}\sim p_{\text{data}}}[\log D(\mathbf{x})] + \\ E_{\mathbf{z}\sim p(\mathbf{z})}[\log(1 - D(G(\mathbf{z})))]\} \end{matrix}, \quad (1)$$

where $D(\mathbf{x})$, $G(\mathbf{z})$ represent the parametrized discriminator and generator respectively. Although GANs have shown great promise in fields such as photorealistic generation and image translation, they are computationally expensive and often limited by training time and instability [4]. Quantum machine learning offers a potential solution to this problem.

Quantum generative adversarial networks (QGANs) leverage the probabilistic nature and high-dimensional parallel processing of quantum computing to achieve significant improvements [5]. Specifically, when the real dataset is both high-dimensional and purely classical, QGANs using quantum processors for the generator and discriminator can achieve an exponential computational advantage over classical GANs [6]. QGAN architectures have been designed and tested on quantum hardware, using IBM-Q quantum processors, for generating images akin to the MNIST dataset of handwritten digits [4]. Previous QGANs have been trained on MNIST, achieving a Hellinger Distance of 0.1951 when 10 parameters each were used for the generator and discriminator respectively [4]. This result was comparable to a classical GAN of 199 trainable parameters, a 94.98% reduction in parameters for a similar quantitative performance [4].

Within the field of image generation, the most important metric of success is whether generated

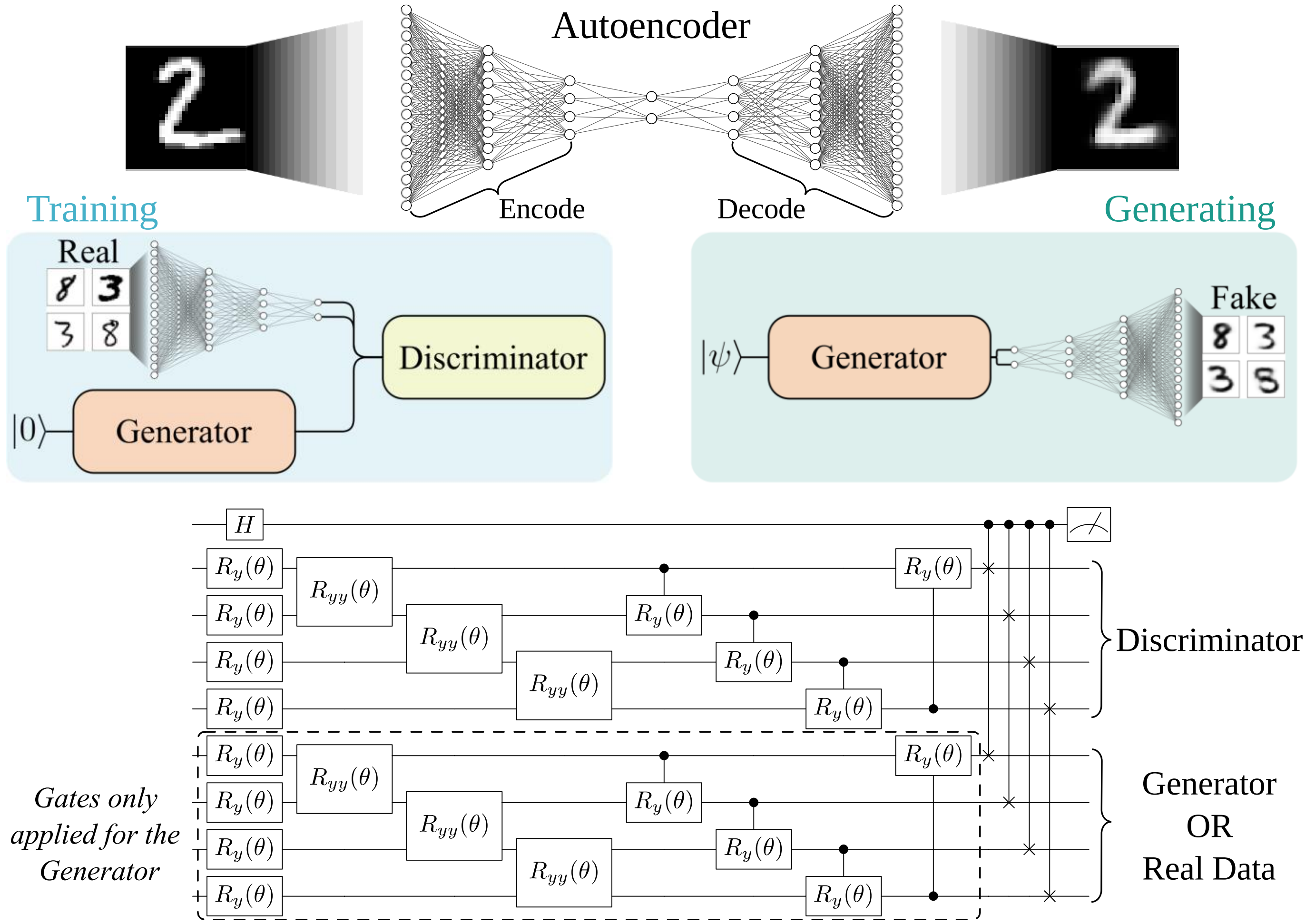

Figure 1: Overview and architecture of the implemented QGAN. A classical autoencoder was trained to downscale, then regenerate, MNIST handwritten digit images. It is then split into an encoder and decoder. The encoder is used during the training procedure to provide real data to the discriminator. The decoder is used following training when the generator creates artificial handwritten digits. The entire quantum circuit for both processes is outlined at the bottom where all parameters are optimized during training.

images appear real to the human eye. Poor generated image quality was the most significant problem of the QGAN implemented by the authors of [4]. This work used PCA to reduce input data from a dimensionality of 784 to 4 for qubit encoding. It is difficult to downscale this dramatically while maintaining fidelity [7]. In fact, the downscaling required to accommodate limited qubits has so far removed any potential quantum advantage [8]. It has been shown, both theoretically and experimentally, that these limitations make PCA a suboptimal dimensionality reduction tool [7].

1.2 Problem Definition

The goal of this project is to improve the quality of generated images by investigating an alternative downscaling technique and implementing it within a QGAN architecture. It is essential that this new method does not prevent the QGAN from learning the distribution of the real data. By successfully training a QGAN to generate images of similar quality to those from the MNIST dataset, the efficacy of these networks can be further demonstrated.

2. METHODOLOGY

This section will describe the various elements of our project, emphasizing the aspects that improve on previous work. Figure 1 shows an overview of the architecture of the implemented QGAN.

2.1 Quantum Circuit

The quantum circuit is composed of the encoding gates, generator, discriminator, and swap test. The encoding gates are R_y gates that rotate a qubit by an angle, $\theta = \arcsin(\sqrt{x})$, which corresponds to a normalized pixel value x. This allows the quantum circuit to take classical data as input. The generator and discriminator are each created with 4 qubits and a series of R_y and R_{yy} gates. Their architectures were designed and justified in previous work and are made similar in complexity so that they can evenly compete until the generator converges. They are both designed to output on all 4 qubits, so they can be compared using a swap test in the final section of the quantum circuit. Here a series of controlled swaps directly compares the similarities between output states, which is quantified by the expectation value of the ancillary qubit. The generator and discriminator are therefore trained to approximate the real data distribution in parallel, however the generator does not have direct access to that data which ensures the uniqueness of its output. This approach is unique to QGANs and significantly differs from the traditional GAN approach where the discriminator classifies received data as real or fake. For more information about the quantum architecture used, see [4].

PennyLane was used to simulate the QGAN's quantum circuit. Previous work used IBM's Qiskit to develop the quantum circuit; however, PennyLane can interface with machine learning libraries such as TensorFlow and PyTorch. This implementation is better suited for future quantum machine learning investigations.

2.2 Dataset and Dimensionality Reduction

A subset of the MNIST handwritten digit dataset was used to train the QGAN model. Each image is represented by a 28x28 array with all indices between 0 and 1. However, training a QGAN to output 784 values would require an infeasible number of qubits for current quantum hardware, necessitating dimensionality reduction of the original images. We refer to the reduced representation as the latent vector of the image. As previously implemented using PCA, the latent vector is generated by the QGAN, then the pixel values of the generated images are constructed as linear combinations of the latent vector [4].

2.3 Autoencoder

Autoencoder neural networks leverage their inherent nonlinearities to enable more sophisticated dimensionality reduction at the expense of complexity. The top of Figure 1 shows such a network, where the first half encodes the input into a low-dimensional space and the second half decodes the latent representation into an inexact copy of the original input. The input and target of an autoencoder are identical. The QGAN is trained to generate 4-dimension latent vectors that can be decoded into a 784-length vector representing the pixel values of a handwritten image. The complete system is a hybrid classical-quantum network that involves the encoder, the QGAN for training, and then the decoder for inference.

3. RESULTS AND DISCUSSION

First, the trained autoencoder was compared qualitatively against the PCA algorithm. MNIST images were reduced to four-dimensional vectors and then returned to the original image using the reverse algorithm. As can be seen in Figure 2, the autoencoder achieves superior qualitative results compared to the PCA algorithm.

Figure 2: sample MNIST images reduced and inverted using an autoencoder versus the PCA algorithm

The QGAN was trained using MINST images reduced by the autoencoder. Testing aimed to verify model learning was preserved using this reduction technique and that generated samples were qualitatively superior. To verify the QGAN was learning the underlying probability distribution independent of the autoencoder, only images labelled with "3" or "8" were provided as training data. Thus, if the QGAN generates only threes and eights it can be concluded that it has properly trained and is not relying on the autoencoder for novel generation.

Images generated by our QGAN are shown in Figure 3. It is clear our results are of a higher quality compared to the images generated by [4]. Our model is similarly qualitatively superior to other state-of-the-art quantum generative adversarial networks.

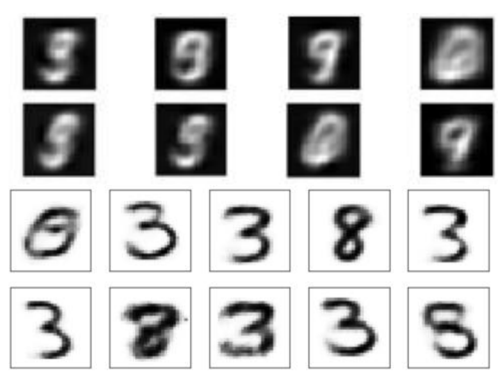

Figure 3: results of [4] *above samples generated by our QGAN. Note that* [4] *trained its QGAN on the subset {4, 6, 9} compared to {3, 8}*

4. CONCLUSIONS AND FUTURE WORK

This paper investigated a hybrid quantum-classical implementation of a GAN. By taking advantage of the superposition and entanglement that is inherent to quantum circuits, the proposed model requires significantly fewer trainable parameters than a similarly capable classical GAN. In this paper, the necessary dimensionality reduction was implemented with a classical autoencoder, which was shown to outperform previous results that used principal component analysis. The MNIST handwritten digit dataset was used in training, with the final generated images appearing to be very similar to the training samples.

Future work should investigate data encoding approaches that leverage superposition to increase the size of the latent vector while still using the same number of qubits. Amplitude embedding coupled with a normalizing layer in the autoencoder is an area of particular interest that was briefly attempted but warrants further investigation. Additionally, while this implementation used the MNIST dataset, dimensionality reduction using an autoencoder is theoretically invariant to the dataset and should be practically implemented on more complex problems.

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